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## **Using game theory to determine the point of interest sharing in a bilateral contract**

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### **Abstract**

The purpose of this research is to protect the interests of the two parties to the contract, who jointly or separately have requirements and restrictions and expect the details of the agreement to be such that their interests are protected. In fact, this research provides a calculation method for agreement in contracts, which is very frequent in the real world and is a bridge to communicate between humanities and computing sciences. Methodology: In this research, a model based on the game theory mechanism is proposed for the equilibrium point. To clarify the details of the contract, the 4p, 7p, ... models that are in the marketing mix are considered. Binary numbers are used to determine game strategies. And finally, a mathematical model is presented to calculate the consequences of the game. Therefore, research is an interdisciplinary work. Findings and results: The number of strategies in such games is exponential in nature, and with the increase in the number of variables, the problem will become very large and difficult, and its solution will be a challenge. In addition to modeling the problem, the solution algorithm of this model is also presented has been done, and real-world examples are given to clarify the issue.

### **Keywords**

Game theory, marketing mix, decision model, bilateral contract

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## **1- Introduction:**

Negotiating in the field of business or financial matters is always associated with anxiety, although negotiation techniques, as a skill, help to better identify opportunities and end the negotiation in our favor. But such an approach may be the answer in simple matters, but in complex transactions, a strategic approach should be considered. Having diverse and flexible strategies along with the use of negotiation techniques helps to achieve better opportunities in the negotiation process and obtain results that lead to providing value for both sides of the transaction. The best form of negotiation will be achieved when the parties, with full understanding of each other's strategies, interact in line with the goal of win-win and reach a common strategy that will provide their common interests at the highest level. In this research, a special type of negotiation has been taken into consideration, in which the parties seek to divide the work in such a way that the division of work is completely balanced. In the real world, there are such negotiations in various fields. Division of labor in the family, division of labor in the workplace, division of labor to go on a trip, emphasizing some expectations and giving up on others in a bilateral contract are all examples where, in order to achieve a goal, Details are defined and each party accepts a part of these details. According to what was explained, the answer to three questions is important to reach a good agreement:

- 1- How to determine the indicators of the parties.
- 2- How can the strategies of the parties to the contract be defined according to the determined indicators.
- 3- How can the consequences of choosing players' strategies be measured and how to determine the optimal strategy of both sides of the contract.

## **2- Theoretical foundations:**

### **2-1- Game theory**

In 1921, a French mathematician named Emile Burrell studied a number of common games in casinos; he believed that the results of these types of games can be predicted in logical ways. Although Burrell was the first person who seriously addressed the subject of games, many historians have attributed the creation of game theory to the Hungarian mathematician John von Neumann. His special attention was on a card game, he formulated the method of bluffing in this game. In 1944, together with Oskar Mungstern, who was an Austrian economist, he wrote the book *Theory of Games and Economic Behavior*. Although this book was written solely for economic applications, its applications in psychology, sociology, politics, warfare, recreational games, and many other fields soon became apparent. Game theory uses mathematical models to analyze the cooperation or competition methods of logical and intelligent beings. Game theory is a branch of applied mathematics that is used in social sciences and especially in economics, biology, engineering, political science, international relations, computer science, marketing and philosophy. Game theory tries to use mathematics to estimate behavior in strategic situations or in a game where the success of a person in choosing depends on the choice of others. The ultimate goal of this knowledge is to find the optimal strategy for the players. The important principle of game theory is the reasonableness of players' behavior. Being rational means that each player only seeks to maximize his profit and each player knows how to maximize his profit, so it will be easy to guess their behavior based on the cost-benefit diagram. Like the chess game, you can guess what the experienced opponent will decide.

Game: When the profit of an entity does not depend only on its own behavior and is affected by the behavior of one or more other entities, and the decisions of others have a positive or negative effect on its profit, a game has been formed between. In every game, two concepts of strategy

and strategic thinking are important. players need to use their skills in the best way, it is necessary to change the result of the game in their favor by thinking about the opponent's game and his decisions. This issue is defined in game theory, strategic thinking.

**Game structure:** Each game consists of three basic parts. Players, strategies and consequences. Players are essentially the decision makers of the game, which can be individuals, companies, governments, etc. Strategies are a set of decisions and actions that each player can take. And finally, each subset of the set of possible actions and decisions is called a consequence.

**Nash Equilibrium:** Each game consists of three basic parts. Players, strategies and consequences. Players are essentially the decision makers of the game, which can be individuals, companies, governments, etc. Strategies are a set of decisions and actions that each player can take. And finally, each subset of the set of possible actions and decisions is called a consequence.

**Nash Equilibrium:** Research in this field is often based on a set of strategies known as game balance. These strategies are basically based on rational rules. The most famous equilibrium is the Nash equilibrium. According to the Nash equilibrium point of view, if we assume that in any mixed strategy game, the players choose their strategies in a logical and reasonable way and seek the maximum profit in the game, at least one strategy to obtain the best result for each player is possible. It is a choice, and if the player chooses another solution, he will not get a better result.

## **2-2- Principles of Negotiation**

Negotiation is a conversation-oriented process that focuses on a specific issue and aims to resolve a dispute or reach common interests. Negotiation can be done for more than two people and it will be successful if both parties benefit from it. Negotiations can be divided into two general categories, good and bad. Good negotiations are negotiations based on maintaining the principles in which the four main pillars are respected. 1- Persons are separated from the subject of negotiation. 2- The focus is on interests, not positions. 3- Negotiation is not limited to one solution and has several solutions. 4- Objective criteria are used instead of subjective criteria. On the other hand, bad negotiations can be divided into three groups. 1- Negotiations that are based on taking a position. In these negotiations, the positions have been determined in advance and the parties are trying to reach their positions instead of reaching their goals. 2- Hard negotiations in which one party has a competitive advantage due to having the tools of power. 3- Soft negotiations, where one side is only looking for an agreement and it is not unlikely that it will stay away from achieving its interests.

## **2-3- Marketing Mix:**

Marketing business is not just one aspect. In fact, it is a mixture of different issues that, if managed together, will help business grow. The set of these issues is called the marketing mix. In 1960, a marketing professor named Edmund Jerome McCarthy proposed the concept of the "marketing mix", and considered a structure for it that he believed included all the marketing aspects of a business: Product. Price, Place and Promotion. Due to the fact that all these words start with the letter P, this structure is called 4P. Twenty years after Mr. McCarthy, a number of marketing theorists concluded that the four-P model is not very complete and does not answer the needs of some



businesses (especially service-oriented companies). Therefore, three more parts were added to the 4p model: People, Physical Evidence, and Process. With the addition of these parts, the shortcomings of the previous model were partially eliminated and (thanks to the commitment of these dear marketers to the letter P) the 4p structure was transformed into a 7p.

- **Product:** quality, specifications, features, brand, after-sales service, etc. are among the things that are important in choosing a product by the consumer.
- **Price:** Two factors are important in this index. pricing should be competitive and with the highest profit.
- **place:** The location and method of accessing the product by the customer is important in this index.
- **Promotion:** communication methods with customers, motivational offers and sets of advertising behaviors are important in this index.
- **People:** the way personnel deal with customers in order to attract satisfaction is considered in this index.
- **Process:** the set of actions aimed at supporting the timely delivery of the product is considered in this index. The ability to provide face-to-face and non-face-to-face services is one of the cases of providing better services.
- **Physical evidence:** the appearance of the company or store is one of the things that can play an effective role in the customer's mind.

#### **2-4- Research history:**

Jon M. & David E. (2014), One of the fundamental approaches to the study of negotiations between marketing members is an examination of the distinctions between the distributive and integrative orientation. Application of this paradigm would be facilitated if channel members were better able to size up a future negotiation to determine which orientation would be appropriate under the current situational factors faced. This article presents the concept of adaptive negotiation as a method to help negotiators select their style given the circumstances faced with channel partners. Kelly, M & Hauck, E. (2015), The field of research is about the division of labor in the family. They suggest that the division of housework in these queer couples is an opportunity to reconstruct gender by challenging normative gender roles and creating alternatives to how gender shapes social life. Game theory has the ability to model problems in different sciences and various research works have been done in this field, including in the economic field, we can refer to the research of Ali Emami et al. (2015) which focuses on the economic modeling of politics. In the field of optimization, Namdi et al. (2015) in their research, while paying attention to customer demand and earning profit, minimizing fuel cost and gas production have considered a greenhouse, in the field of energy, Yugwa et al. (2018) have introduced a game theory approach to reduce the environmental risk of companies in their research. Bennett (1995) by bringing many evidences and studies showed that game theory is an ideal approach for modeling in the space of international communication and in which every country tries to maximize its interests in this communication. In this article, he concludes that game theory, because it uses the rules of intelligent human choice for modeling, is more reliable compared to other competing modeling models. Jun Du & Roger J. Jiao (2022), This paper examines the critical issues and research opportunities for establishing a theoretical foundation of value-driven multiplayer game decision making in large complex engineering and systems design projects. This paper envisions a research framework for value-driven cooperative negotiation by synthesising the mathematical foundations of utility theory. Daofei Li & Hao Pan (2022), Overtaking on two-lane two-way (TLTW) highways is often associated with a high risk

of crashing. For accurate overtaking modelling, it is crucial to consider the uncertainties of interacting vehicle behaviours, especially their driving styles. To address these needs, this research proposes a TLTW overtaking decision model using a level-k game theoretic framework, which can consider the mutual influences between the ego and oncoming vehicles of different driving styles. Haojia He, Hua Tang & Songbo Guo (2022), This paper considers the relative pay-offs amongst two manufacturers' selection for their respective scarce technology of the Internet of Things, using game theory models and numerical analysis. The findings provide implications for competitive enterprises in high-tech industries to acquire complementary technologies by identifying what conditions lead to win-win outcomes. In particular, firms can adjust their licensing levels under certain conditions to adapt to the disparities in each other's development capabilities, thus achieving cross-licensing agreements. Mousapour Mamoudan, M et al. (2022), determining a reasonable price requires a precise pricing strategy. In this paper, a pricing model for perishable food is presented in which the brand value of the product and the price of other manufacturers as competitors are considered. In the coordination contract game-theory model, Multi Retailer- one Supplier and Price-sensitive demand of Perishable product are developed with and without quantity discount contract. Obtained results indicate that independent procurement provides retailers with higher profit, while lower profit will be presented when coordination is not considered. Also, with coordination, the ordering cycle increases, and the ordering frequency decrease.

### **3- problem statement**

Two negotiators intend to agree on an issue. This issue has different dimensions and it can be divided into several indicators. The negotiating parties are trying to reach a balance point by giving points and taking points in each of these indicators, so that the interests of both parties are met equally. The parties can have a unique index that is unimportant for the other party or an index that has the same or variable importance for both parties.

#### **3-1- Statement of the problem in the form of game theory**

1 -Players: each negotiating party is a player, and  $x$  and  $y$  variables are considered for them.

2 -Strategies: preserving or not preserving the interests of indices. In each index, if a player puts it in his priorities. The value of that index will be one and this index will be removed from the other party's priorities and will be zero. In fact, the right to choose the conditions in this index is given to one of the players and the other player will accept the conditions of this player.

3 -Consequences of the game: The maximum value of the linear combination is the result of multiplying the importance coefficient of each index by the value of the index.

How to calculate game outcomes

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*i*: Index counter

*j*: Strategy counter

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*w<sub>i</sub>*: Importance coefficient of index *i* for negotiator *x*

*v<sub>i</sub>*: Importance coefficient of index *i* for negotiator *y*

*x<sub>ij</sub>*: The value that negotiator *x* chooses for index *i* and for strategy *j*

*y<sub>ij</sub>*: The value that negotiator *y* chooses for index *i* and for strategy *j*

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$\max_j \min \left( \sum_{i=1}^n w_i x_{ij}, \sum_{i=1}^n v_i y_{ij} \right)$	(1)
s.t:	
$\sum_{i=1}^n w_i x_{ij} - \sum_{i=1}^n v_i y_{ij} < \varepsilon \text{ for } j = 1, \dots, m$	(2)
$x_{ij} \neq y_{ij} \text{ for } j = 1, \dots, m, \text{ for } i = 1, \dots, n$	(3)
$x_{ij} = \{0,1\} \text{ for } j = 1, \dots, m, \text{ for } i = 1, \dots, n$	(4)
$y_{ij} = \{0,1\} \text{ for } j = 1, \dots, m, \text{ for } i = 1, \dots, n$	(5)

**3-2- Further Details:**

Let's assume that a contract is to be drawn up between two people, and each party has three indicators, and out of these three indicators, two indicators are common to the parties. Therefore, there are 4 indices in total, the first index is the exclusive index of actor 1, and will always have a value of one for actor 1 and a value of zero for the second actor. the second index is an exclusive index for the second actor, and always has a value of one for the second actor, and will have a value of zero for the first actor. Indexes 3 and 4 can have zero and one values alternately between two actors. Therefore, a total of 4 strategies can be presented for the game.

Table 1- Table of game strategies when there are two common indicators for the parties to the contract.

x					y			
P1	P2	P3	P4		P1	P2	P3	P4
1	0	0	0	Strategy1	0	1	1	1
1	0	0	1	Strategy2	0	1	1	0
1	0	1	0	Strategy3	0	1	0	1
1	0	1	1	Strategy4	0	1	0	0

The mechanism of the total number of indicators and strategies follows a general rule. The number of indicators will always be equal to the sum of the indicators of the parties minus the common indicators, and the number of strategies for each player will be equal to the number of 2 to the power of the number of common indicators.

$$total\ indicators = N(indicators(x)) + N(indicators(y)) - N(indicators(x) \cap N(indicators(y)))$$

$$total\ strategies(x) = total\ strategies(y) = 2^{N(indicators(x) \cap N(indicators(y)))}$$

In the described example, we will have:

$$total\ indicators = 3 + 3 - 2 = 4$$

$$total\ strategies(x) = total\ strategies(y) = 2^2 = 4$$

**3-3- Practical example:**

The symbol x is a company that supplies health products, and it intends to set up a contract with a store with the symbol y to sell its products. The contract amount is a percentage of the sales share that the parties have agreed upon. In order to interact regarding the details of the contract, the parties should plan in such a way that out of the 7 main factors in marketing, the priority of selection and targeting for its implementation will be divided according to their requirements. In such a way that the authority to plan and implement each of the 7p will be given to only one of the parties. Determining the equilibrium point of this contract using game theory is as follows.

1- Players: player1=x and player2=y

2- Strategies:

- Product: It is a specific indicator for the product supplier, inherently gives the priority in providing products from actor x to actor y. Therefore, the value of this variable will always be 0 for x and 1 for y.

- Price: The store is more inclined to price with competitive criteria, because less time will be spent to store the product and the probability of selling the product will increase in less time. On the other hand, the product supplier tends to be more profitable. Therefore, both sides of the contract, tend to be responsible for policy making in this index.

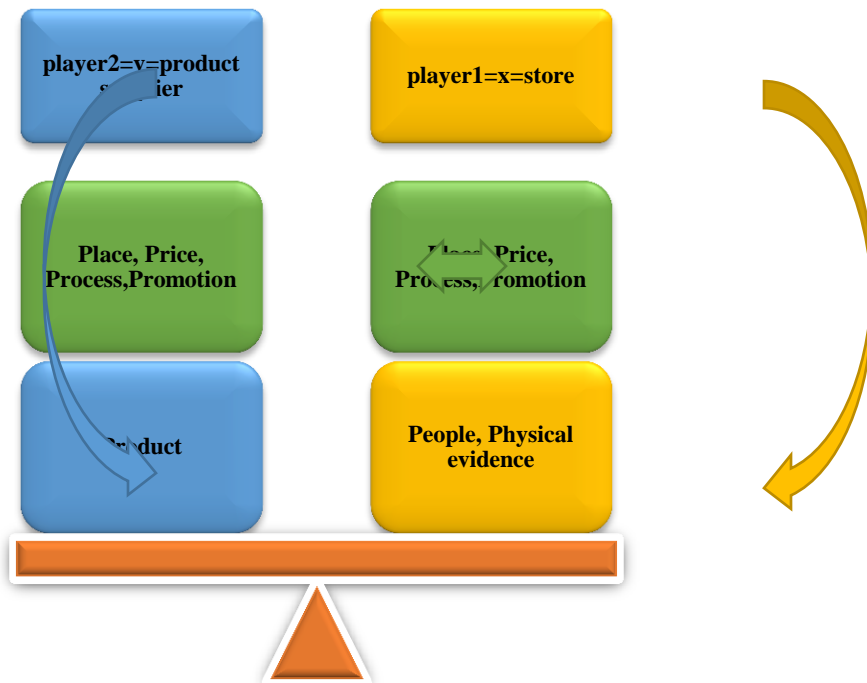
- Place: In this index, the negotiation of the parties is to get the right to choose the placement of the product in the store. In every store, there are points with a better access level that can attract the attention of the customer more, therefore, the parties to the contract tend to have the right to place the product. This index is also a common index. Promotion: communication methods with customers, motivational offers and sets of advertising behaviors are important in this index.

- Promotion: In this index, it is expected that through the creation of motivational plans and advertisements, the customer will be attracted to the product and intend to buy it. The parties tend to implement their ideas in this field and it is one of the common indicators.

- People: The selection of store personnel is one of the store's specialty indicators, and practically, the store has the priority in this indicator.

- Process: In this index, attention has been paid to answering absentee orders. It is important to send absentee orders through the store or Rasa by the product supplier. Therefore, it is important for the parties to have priority in this index.

- Physical evidence: In this index, attention has been paid to answering absentee orders. It is important to send absentee orders through the store or Rasa by the product supplier. Therefore, it is important for the parties to have priority in this index.



It is obvious that each store and each product supplier has a different weight for the defined indicators. For the proposed model of this research, the proportion of weight between indicators was obtained from several stores and product suppliers and used in game strategies. Table 2 presents the weighted data and total strategies of the game.

Table 2- Weights and strategies of a contract with a total of 7 indicators, 4 of which are common.

$w_i$ : The coefficient of importance of indicators for the store							$v_i$ : The coefficient of importance of indicators for the product supplier						
$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$	$w_7$	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$v_7$
0.37	0.05	0.11	0.18	0.14	0.07	0.08	0.05	0.21	0.15	0.24	0.09	0.08	0.18
$\bar{X}_j$ : Store strategies $\bar{X}_j = [x(1), \dots, x(7)]$ for $j = 1, \dots, 2^4$							$\bar{Y}_j$ : product supplier strategies $\bar{Y}_j = [y(1), \dots, y(7)]$ for $j = 1, \dots, 2^4$						
P1	P2	P3	P4	P5	P6	P7	P1	P2	P3	P4	P5	P6	P7
0	0	0	0	1	0	1	1	1	1	1	0	1	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	0	1	1	1	1	0	0	1	0
0	0	0	1	1	1	1	1	1	1	0	0	0	0
0	0	1	0	1	0	1	1	1	0	1	0	1	0
0	0	1	0	1	1	1	1	1	0	1	0	0	0
0	0	1	1	1	0	1	1	1	0	0	0	1	0
0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	1	0	0	1	0	1	1	0	1	1	0	1	0
0	1	0	0	1	1	1	1	0	1	1	0	0	0
0	1	0	1	1	0	1	1	0	1	0	0	1	0
0	1	1	0	1	1	1	1	0	0	1	0	0	0
0	1	1	0	1	1	1	1	0	0	1	0	0	0
0	1	1	1	1	0	1	1	0	0	0	0	1	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0

3- Consequences of the game: In order to calculate the results of the game and the equilibrium point between the store and the product supplier, calculations were made using the model presented in the research and the results are presented in Table 3.

Table 3- Calculating the result of the strategies in Table 2 and introducing the best strateg

	strategy															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\sum w_i \cdot \bar{X}_j$	0.22	0.29	0.40	0.47	0.33	0.40	0.51	0.58	0.27	0.35	0.45	0.53	0.38	0.45	0.56	0.63
$\sum w_i \cdot \bar{Y}_j$	0.73	0.65	0.49	0.41	0.58	0.49	0.34	0.26	0.52	0.44	0.28	0.20	0.37	0.29	0.13	0.05
<b>min</b>	<b>0.22</b>	<b>0.29</b>	<b>0.40</b>	<b>0.41</b>	<b>0.33</b>	<b>0.40</b>	<b>0.34</b>	<b>0.26</b>	<b>0.27</b>	<b>0.35</b>	<b>0.28</b>	<b>0.20</b>	<b>0.37</b>	<b>0.29</b>	<b>0.13</b>	<b>0.05</b>
	<b>max = 41</b>															

The results show that strategy number 4, in which the product supplier has the right to choose 3 indicators, Product-Price-place, and the store has the right to choose Promotion-People-Process-



Physical evidence indicators, is a balanced and win-win strategy. In fact, the product supplier will have the right to determine the type of product, the price of the product, and the location of the product in the store, and on the other hand, the store management, according to its policy, is allowed to choose 4 indicators. These indicators include personnel - appearance - support for non-attendance services - incentive and advertising program. Table 4 shows the numerical values of what was explained.

Table 4 - Show the details of the best strategy

$\bar{X}_4$ : Store strategies							$\bar{Y}_4$ : product supplier strategies						
P1	P2	P3	P4	P5	P6	P7	P1	P2	P3	P4	P5	P6	P7
0	0	0	1	1	1	1	1	1	1	0	0	0	0
$W \cdot \bar{X}_4$							$W \cdot \bar{Y}_4$						
0.00	0.00	0.00	0.18	0.14	0.07	0.08	0.05	0.21	0.15	0.00	0.00	0.00	0.00
$\Sigma W \cdot \bar{X}_4 = 0.47$							$\Sigma W \cdot \bar{Y}_4 = 0.41$						

**4- Algorithm for solving big problems:**

In a situation where the number of decision-making indicators is high, solving the problem with the mentioned description will face a challenge and will require the use of computing software with coding capabilities. To clarify the discussion, in Table 9, the number of strategies of each negotiator is shown by increasing the number of indicators, considering that the number of strategies is an exponential function, in negotiations where the number of decision indicators is 20. reach, there are more than a million strategies, and if the number of indicators reaches 50, there will be more than a trillion strategies for each side. For this purpose, an algorithm has been provided to facilitate the calculation process.

Table 5- Increasing the number of strategies by increasing the number of indicators

$2^1$	$2^2$	$2^3$	$2^4$	$2^5$	$2^{10}$	$2^{20}$	$2^{30}$	$2^{40}$	$2^{50}$
2	4	8	16	32	1,024	1,048,576	1,073,741,824	1,099,511,627,776	1,125,899,906,842,624

Step 1- Assume variable x for the first player and variable y for the second player.

Step 2- The negotiation indicators should be classified into three categories.

- Specific indicators of player 1 with the symbol  $S_x$
- Specific indicators of player 2 with the symbol  $S_y$
- Shared indicators between two players with symbols  $S_{x \cap y}$

Step 3- Determining the coefficient of importance of the indicators for each of the players, which are placed in two vectors with the corresponding symbols  $w_x$  and  $w_y$

Step 4- Generating the table of game strategies as follows:

- For player 1: The following matrix is generated to represent the strategies.

$$\left[ \begin{matrix} \left( \begin{matrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{matrix} \right)_{2^{n(S_{x \cap y})} * n(S_x)} & \left( \begin{matrix} \text{The number 1 is in binary} \\ \vdots \\ \text{The number } 2^{n(S_{x \cap y})} \text{ is in binary} \end{matrix} \right) & \left( \begin{matrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{matrix} \right)_{2^{n(S_{x \cap y})} * n(S_y)} \end{matrix} \right]$$

- For player 2: The following matrix is generated to represent the strategies.

$$\left[ \begin{matrix} \left( \begin{matrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{matrix} \right)_{2^{n(S_{x \cap y})} * n(S_x)} & \left( \begin{matrix} \text{The number } 2^{n(S_{x \cap y})} \text{ is in binary} \\ \vdots \\ \text{The number 1 is in binary} \end{matrix} \right) & \left( \begin{matrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{matrix} \right)_{2^{n(S_{x \cap y})} * n(S_y)} \end{matrix} \right]$$

Example: If there are three common indicators, and there is a specific indicator for each player, the following matrix is generated.

$$\text{For player 1: } \begin{bmatrix} 1 & 000 & 0 \\ 1 & 001 & 0 \\ 1 & 010 & 0 \\ 1 & 011 & 0 \\ 1 & 100 & 0 \\ 1 & 101 & 0 \\ 1 & 110 & 0 \\ 1 & 111 & 0 \end{bmatrix} \quad \text{For player 2: } \begin{bmatrix} 0 & 111 & 1 \\ 0 & 110 & 1 \\ 0 & 101 & 1 \\ 0 & 100 & 1 \\ 0 & 011 & 1 \\ 0 & 010 & 1 \\ 0 & 001 & 1 \\ 0 & 000 & 1 \end{bmatrix}$$

Step 5- Calculate the consequences of each game strategy for the parties.

- The weight vector obtained for each player is multiplied by the players' strategies matrix.
- Calculate the row sum for the generated matrix.:

Step 6- The Nash equilibrium of the game should be calculated by the min-max method as follows.

- Enter the number of strategies in column 1 of the new matrix.
- The result of player 1's strategies should be inserted in column 2 of the new matrix.
- The result of player 2's strategies should be entered in column 3 of the matrix.
- In column 4 of the matrix, values minimize of columns 2 and 3 be calculated.
- In column 1 of the matrix, the row number corresponding to the maximum value calculated in column 4 is the balanced strategy for the parties.

#### 4-1- A practical example with large dimensions

Two tourists are planning to go on a trip and a list of necessary things has been prepared for this trip. There are 25 defined tasks in the list and it is decided to divide the tasks according to the needs and capabilities of the parties. With a simple simulation, two players can be considered in game theory, where there are 25 indicators and given to the parties with priority. For the first player, it is not possible to perform 5 indicators, and for the second player, it is not possible to perform 3 indicators. The remaining 17 indicators are proportional to the ability of two players with different weights as follows.

Table 6- Weights assigned to the indicators for the parties to the contract

indicator s	1	2	3	4	5	6	7	8	9	10	11	12	13
weights	w1	w2	w3	w4	w5	w6	w7	w8	w9	w1	w1	w1	w1
player1	∞	∞	∞	∞	∞	1.4	0.3	1.4	2.93	7.2	11.	0.2	2.8
player2	7.54	1.5	2.58	2.6	6.1	6	7.0	4.9	2.82	4.2	4.2	0.8	4.6
		4		1	4	2.5	4	3		1	6	1	5
indicator s	14	15	16	17	18	19	20	21	22	23	24	25	
weights	w14	w1	w16	w1	w1	w1	w2	w2	w22	w2	w2	w2	
player1	4.81	4.4	12.0	0.6	3.0	6.3	2.9	2.4	11.8	8.9	3.7	0.3	
player2	6.98	5.1	6.46	2.5	5.3	7.4	4.7	2.5	7.30	∞	∞	∞	
		6	8	7	4	0	1	5	4	1	2	1	
		0		4	1	1	6	1					

**4-2- Problem Solving:**

Step 1-  $x = \text{player1}$   $y = \text{player2}$

Step 2-  $S_x = \{\text{indicator23}, \text{indicator24}, \text{indicator25}\}$

$S_y = \{\text{indicator1}, \dots, \text{indicator5}\}$

$S_{x \cap y} = \{\text{indicator6}, \dots, \text{indicator22}\}$

Step 3- In the issue of information entered.

Step 4- Generating the table of game strategies as follows:

- For player 1: The following matrix is generated to represent the strategies.

$$\left[ \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_{2^{17} \times 5} \quad \begin{pmatrix} \text{The number 1 is in binary} \\ \vdots \\ \text{The number } 2^{17} \text{ is in binary} \end{pmatrix} \quad \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{pmatrix}_{2^{17} \times 3} \right]$$

- For player 2: The following matrix is generated to represent the strategies.

$$\left[ \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{pmatrix}_{2^{17} \times 5} \quad \begin{pmatrix} \text{The number } 2^{17} \text{ is in binary} \\ \vdots \\ \text{The number 1 is in binary} \end{pmatrix} \quad \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_{2^{17} \times 3} \right]$$

Step 5,6- Due to the large amount of data, Excel and MATLAB software can be used. In this research, the BASE (number,2,17) function, in Excel software is used to generate game strategies for common indicators. Then, other functions of this software, which are widely used, have been used for the required calculations. The calculation results are listed in Table 7. Because the acceptance of the work by each of the parties will naturally entail costs for them, the name "cost" has been used for a better understanding of the issue.

Table 7- Consequence details of the best strategy

indicate	1	2	3	4	5	6	7	8	9	10	11	12	13
cost	cost 1	cost 2	cost 3	cost 4	cost 5	cost 6	cost 7	cost 8	cost 9	cost 10	cost 11	cost 12	cost 13
player1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.93	7.20	11.36	0.00	0.00
player2	7.55	1.54	2.58	2.61	6.14	2.50	7.04	4.93	0.00	0.00	0.00	0.81	4.65
indicate	14	15	16	17	18	19	20	21	22	23	24	25	
cost	cost 14	cost 15	cost 16	cost 17	cost 18	cost 19	cost 20	cost 21	cost 22	cost 23	cost 24	cost 25	
player1	4.81	0.00	12.08	0.00	0.00	0.00	0.00	2.45	11.84	8.91	3.72	0.31	
player2	0.00	5.10	0.00	2.54	5.31	7.41	4.76	0.00	0.00	0.00	0.00	0.00	

By observing the results in Table 7, it is quite evident that in total 10 defined tasks for the trip will be undertaken by passenger1, and 15 defined tasks will be done by passenger2.

In total, 13,1072 strategies, which are the result of choosing different combinations of 17 common indicators, have been identified, and finally, the 14659th strategy has been able to have the best cost balance for the parties to the agreement. Figure 1 shows a representation of the division of duties of the two parties requesting the trip, along with the cost it brings for each.

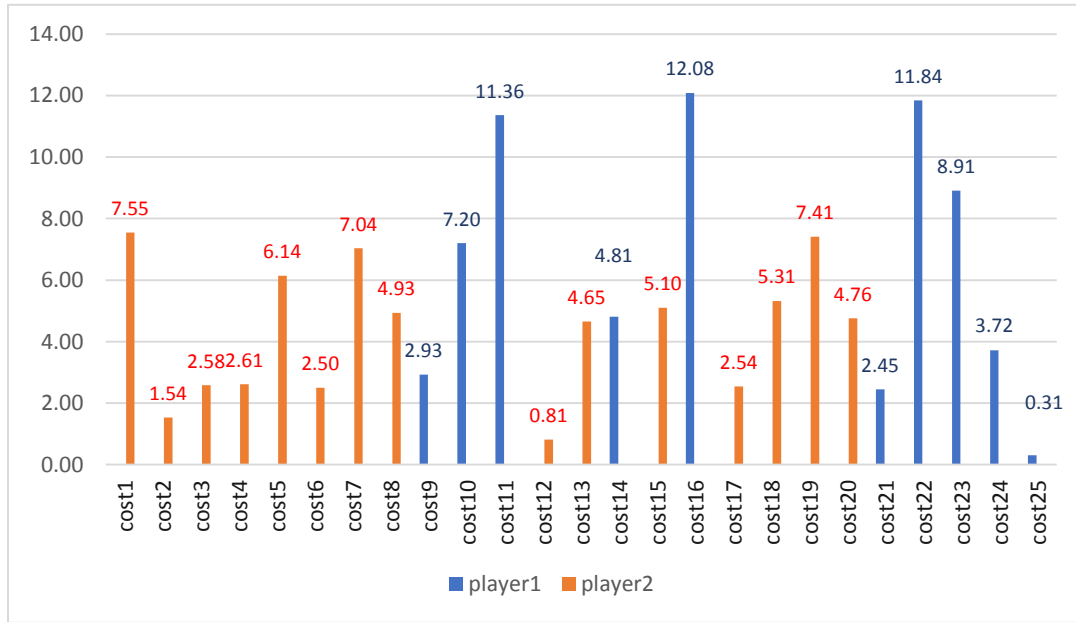


Figure 1- Work division diagram between two passengers

### Conclusion:

The purpose of this research is to model decision-making in the conditions of negotiations between two parties who intend to agree on a work contract. Any contract whose components can be separated and the implementation of its components can be left to the parties, is decided by using the modeling presented in this research. Since game theory has a structure that implements qualitative concepts in the form of mathematical models, it has become the basis of research modeling. In the following, to clarify the issue, the questions in the introduction section will be answered in detail.

One of the necessary measures for modeling is determining decision indicators. In order to use a scientifically accepted method, this research has proposed to use the marketing mix method to achieve the details of the agreement. In fact, performing contractual activities between the parties is considered a service that can be simulated with 4p and 7p methods.

After determining the decision-making indicators, it is necessary to determine the decision-making strategies. For this purpose, the ability of numbers in base 2 is used, which turns an indicator light on for one party and off for the other.

There are a variety of strategies for the parties, each of which will have consequences. Finding the outcome equilibrium point for the parties is a matter determined using game theory capabilities.

In the research, the relative valuation of the indicators is in the contracted weights, and the limitations and requirements of the parties of a contract are considered as strategies with values of zero and one. These have been the researcher's idea for modeling, and changing them can be the subject of future research.

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