International Journal of

Business Management and Entrepreneurship

Developing a Fuzzy Network DEA Model with Linking Constraints to Evaluate Overall and Stage-Level Efficiency of Supply Chains under Uncertainty: An Application in Automotive Manufacturing

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Received: 2026/01/30 Accepted: 2026/02/20 Published: 2026/06/20

ABSTRACT

Traditional data envelopment analysis (DEA) models for assessing supply chain efficiency treat stages independently, ignore links between them, and overlook data uncertainty. This study presents a fuzzy network data envelopment analysis (FNDEA) model to assess the overall efficiency of a three-stage supply chain (supplier, manufacturer, and distributor) under uncertainty. Input, output, and intermediate product variables are modeled as triangular fuzzy numbers. To solve the fuzzy model, the α-cut approach is used, transforming it into two deterministic linear programs with lower (optimistic) and upper (pessimistic) bounds. By applying compatibility constraints on the flow of intermediate products, the model simultaneously accounts for dependencies between stages and presents the efficiency of each stage and the overall efficiency of the chain as intervals. The model is implemented on data from 10 real supply chains, and efficiency is calculated at five α-cut levels of 0, 0.25, 0.5, 0.75, and 1. The results show that the proposed model identifies inefficient steps and provides improvement paths at different levels of uncertainty. This approach allows managers to base decisions on efficiency intervals rather than definite numbers and to conduct more robust analyses in uncertain environments.

Keywords

Network Data Envelopment Analysis, Fuzzy Logic, Supply Chain Efficiency Assessment, Alpha-Cut Approach, Uncertainty

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1. Introduction

Supply chain management is known as the backbone of all industries, as it guides the flow of products and services from producer to consumer. Today, supply chain management faces more challenges due to globalization, rising customer expectations, and process complexity. Current supply chain models, especially those that rely on manual processes and stand-alone databases, are not able to adequately respond to these challenges (Jha et al,2022). These limitations arise from problems such as a lack of real-time inventory visibility, inefficient warehouse management, delayed shipments, and high operating costs, among others. In addition, revenue pressures and the need for flexibility in a market with changing customer expectations intensify these challenges (Abualigah et al., 2023). Data envelopment analysis (DEA), first introduced by Charnes et al. (1978), is a well-established method for measuring the relative efficiency of decision-making units (DMUs) and is now recognized as an effective framework for optimizing collaboration strategies in supply chain management. Beyond traditional DEA, InDEA extends this method by specifying the necessary changes in inputs and outputs to achieve a targeted level of efficiency, especially for dynamic and goal-driven collaborations (Wei et al., 2000). The effectiveness of DEA in enhancing efficiency-based collaborations among supply chain members has been confirmed in several studies, including Kohers et al. (2000), Guijarro et al. (2020), and Amin and Boamah (2020).

DEA is a nonparametric, nonstatistical method that "makes no assumptions about the distribution of inefficiencies or the shape of the production function." However, it is subject to some technical limitations such as uniformity and convexity (Jauhar & Pant, 2016). This feature is considered one of the main advantages of DEA over parametric methods. DEA is data-driven, capable of transforming multiple inputs into multiple outputs, and is used to evaluate parallel decision-making units (DMUs) (Jauhar et al, 2015). In contrast to parametric methods that "focus on means and deviations," DEA focuses on maximum observations. Furthermore, DEA can simultaneously utilize multiple inputs and outputs, whereas parametric methods are usually limited to a single output (Karimi et al,2020). This feature makes DEA "a very suitable data-driven tool for efficiency analysis" and "a useful method for performance evaluation and decision-making" (Karimi et al.,2022). DEA identifies relatively efficient units in the sample and establishes an efficiency frontier. Other DMUs measure the inefficiency of their inputs or outputs by comparing them with efficient units. Therefore, DEA is also considered a frontier econometric method. Units on the frontier have the best relative efficiency, and those that are inefficient fall below the frontier (Kaur & Singh,2021). Also, DEA results usually range from 0 to 1 (or 0 to 100%); a value of 1 indicates relative efficiency, and values less than 1 indicate relative inefficiency. This feature allows for easy comparison of DMUs in the sample (Kumar et al, 2014; Kumar et al, 2016).

The two main DEA models are named after their founders: the CCR model (Charnes, Cooper, and Rhodes) and the BCC model (Bunker, Charnes, and Cooper) (Cikovi´c et al.,2022). The CCR model operates on the assumption of "constant returns to scale" (CRS), meaning that "outputs increase in proportion to inputs" (Cooper et al., 2006). In contrast, the BCC model employs the assumption of "variable returns to scale" (VRS), meaning that proportional changes in inputs do not necessarily lead to proportional changes in outputs. Visually, the CCR model is represented as a straight line, while the BCC model is represented as a convex hull (Hodži´c & Alibegovi´c, 2019). However, it should be remembered that DEA is a method for examining relative (not absolute) efficiency among equal decision-making units in the sample (Giustiniani & Ross, 2008). The limitations and weaknesses of DEA have been the subject of much scientific research, but most researchers believe that "its benefits outweigh its limitations" and that DEA should be considered a "valuable diagnostic tool" (Cikoviˇc & Lozi´c., 2022).

1.1. Research Gap

Traditional DEA models evaluate supply chain stages independently, ignore the relationships and dependencies between stages, and assume that the data is accurate and error-free. This study seeks to fill this gap by developing a Fuzzy Network DEA (FNDEA) model. The proposed model is able to consider the dependencies between different stages of the supply chain, incorporate data uncertainty into the analysis through fuzzy logic, and ultimately evaluate the overall efficiency of the integrated supply chain.

1.2. Supply Chain Structure

In this study, a three-stage supply chain is considered in series, including supplier, manufacturer, and distributor. In this structure, external inputs enter each stage, intermediate products flow between stages, and finally, the final output exits the last stage.

2. Mathematical Model Development

In this section, the necessary notations are explained first, and then the mathematical structure of the proposed Fuzzy Network Data Envelopment Analysis (FNDEA) model for a three-stage supply chain is presented. This model is designed to measure the efficiency of each stage independently and to evaluate the overall efficiency of supply chains under uncertainty, modeling inputs, outputs, and intermediate products as triangular fuzzy numbers.

2.1. Notation

The following indicators, parameters, and decision variables are used throughout the paper.

Sets & Indices

- $j = 1,2,\dots,n$: Decision Making Units (Supply Chains)
- $k = 1,2,3$: Stages (S, M, D)
- $i = 1,\dots,m_k$: Inputs to stage k
- $r = 1,\dots,s_k$: Outputs from stage k
- $p = 1,\dots,q_{k,k+1}$: Intermediate products from stage k to $k+1$

Fuzzy Parameters

- $\tilde{x}_{ij}^{k}$: Fuzzy input i to stage k of DMU j
- $\tilde{y}_{rj}^{k}$: Fuzzy output r from stage k of DMU j
- $\tilde{z}_{pj}^{(k,k+1)}$: Fuzzy intermediate product p from stage k to $k+1$
- $\tilde{\theta}_j^{k}$: Fuzzy efficiency of stage k for DMU j
- $\tilde{\Theta}_j$: Fuzzy overall efficiency of DMU j

2.2. Fuzzy Number Representation

Triangular fuzzy numbers are used:

$$\tilde{a} = (a^L, a^M, a^U) \qquad (1)$$

where:

- a^L: Most pessimistic value
- a^M: Most likely value
- a^U: Most optimistic value

3. Comprehensive Fuzzy Network DEA Model

3.1. Stage-wise Efficiency Models

Stage 1 (Supplier) Efficiency

$$\tilde{\theta}_j^{1} = \max \frac{\sum_{r=1}^{s_1} u_r^1 \, \tilde{y}_{rj}^{1}}{\sum_{i=1}^{m_1} v_i^1 \, \tilde{x}_{ij}^{1} + \sum_{p=1}^{q_{0,1}} w_p^{(0,1)} \, \tilde{z}_{pj}^{(0,1)}} \qquad (2)$$

subject to:

$$\frac{\sum_{r=1}^{s_1} u_r^1 \, \tilde{y}_{rj'}^{1}}{\sum_{i=1}^{m_1} v_i^1 \, \tilde{x}_{ij'}^{1} + \sum_{p=1}^{q_{0,1}} w_p^{(0,1)} \, \tilde{z}_{pj'}^{(0,1)}} \le 1, \forall j' \qquad (3)$$

$$u_r^1, v_i^1, w_p^{(0,1)} \ge \varepsilon \qquad (4)$$

Stage 2 (Manufacturer) Efficiency

$$\tilde{\theta}_j^{2} = \max \frac{\sum_{r=1}^{s_2} u_r^2 \, \tilde{y}_{rj}^{2}}{\sum_{i=1}^{m_2} v_i^2 \, \tilde{x}_{ij}^{2} + \sum_{p=1}^{q_{1,2}} w_p^{(1,2)} \, \tilde{z}_{pj}^{(1,2)}} \qquad (5)$$

subject to:

$$\frac{\sum_{r=1}^{s_2} u_r^2\, \tilde{y}_{rj'}^2}{\sum_{i=1}^{m_2} v_i^2\, \tilde{x}_{ij'}^2 + \sum_{p=1}^{q_{1,2}} w_p^{(1,2)}\, \tilde{z}_{pj'}^{(1,2)}} \preceq 1, \forall j' \qquad (6)$$

$$u_r^2, v_i^2, w_p^{(1,2)} \geq \varepsilon \qquad (7)$$

Stage 3 (Distributor) Efficiency

$$\tilde{\theta}_j^3 = \max \frac{\sum_{r=1}^{s_3} u_r^3\, \tilde{y}_{rj}^3}{\sum_{i=1}^{m_3} v_i^3\, \tilde{x}_{ij}^3 + \sum_{p=1}^{q_{2,3}} w_p^{(2,3)}\, \tilde{z}_{pj}^{(2,3)}} \qquad (8)$$

subject to:

$$\frac{\sum_{r=1}^{s_3} u_r^3\, \tilde{y}_{rj'}^3}{\sum_{i=1}^{m_3} v_i^3\, \tilde{x}_{ij'}^3 + \sum_{p=1}^{q_{2,3}} w_p^{(2,3)}\, \tilde{z}_{pj'}^{(2,3)}} \preceq 1, \forall j' \qquad (9)$$

$$u_r^3, v_i^3, w_p^{(2,3)} \geq \varepsilon \qquad (10)$$

3.2. Overall Supply Chain Efficiency Model

To account for interdependencies between stages, we propose a centralized model. This model maximizes the ratio of the sum of all final outputs to the sum of all primary and intermediate inputs, while ensuring that intermediate products are consistently and equally accounted for by both the supplying and receiving stages.

Centralized Model with Linking Constraints

$$\tilde{\Theta}_j = \max \frac{\sum_{k=1}^{3} \sum_{r=1}^{s_k} u_r^k\, \tilde{y}_{rj}^k}{\sum_{k=1}^{3} \left(\sum_{i=1}^{m_k} v_i^k\, \tilde{x}_{ij}^k + \sum_{p=1}^{q_{k-1,k}} w_p^{(k-1,k)}\, \tilde{z}_{pj}^{(k-1,k)} \right)} \qquad (11)$$

Subject to:

1. Stage Efficiency Constraints:

$$\frac{\sum_{r=1}^{s_k} u_r^k\, \tilde{y}_{rj'}^k}{\sum_{i=1}^{m_k} v_i^k\, \tilde{x}_{ij'}^k + \sum_{p=1}^{q_{k-1,k}} w_p^{(k-1,k)}\, \tilde{z}_{pj'}^{(k-1,k)}} \preceq 1, \forall j', \forall k \qquad (12)$$

2. Intermediate Product Flow Consistency:

$$\sum_{j'=1}^{n} \lambda_{j'}^k\, \tilde{z}_{pj'}^{(k,k+1)} = \tilde{z}_{pj}^{(k,k+1)}, \forall p, \forall k = 1,2 \qquad (13)$$

$$\sum_{j'=1}^{n} \lambda_{j'}^{k+1}\, \tilde{z}_{pj'}^{(k,k+1)} = \tilde{z}_{pj}^{(k,k+1)}, \forall p, \forall k = 1,2 \qquad (14)$$

3. Free Linkage Activities:

$$\sum_{j'=1}^{n} \lambda_{j'}^k\, \tilde{x}_{ij'}^k \preceq \tilde{x}_{ij}^k, \forall i, \forall k \qquad (15)$$

$$\sum_{j'=1}^{n} \lambda_{j'}^k\, \tilde{y}_{rj'}^k \succeq \tilde{y}_{rj}^k, \forall r, \forall k \qquad (16)$$

4. Non-negativity & Non-Archimedean Constraints:

$$\lambda_{j'}^{k} \geq 0, \forall j', \forall k \qquad (17)$$

$$u_r^k, v_i^k, w_p^{(k-1,k)} \geq \varepsilon > 0, \forall r, i, p, k \qquad (18)$$

5. Variable Returns to Scale (Optional):

$$\sum_{j'=1}^{n} \lambda_{j'}^{k} = 1, \forall k (\text{for VRS version}) \qquad (19)$$

3.3. Fuzzy Relation Definitions

Fuzzy Inequality (⪯):

$$\tilde{a} \preceq \tilde{b} \Rightarrow \begin{cases} a^U \leq b^L & \text{(pessimistic)} \\ a^M \leq b^M & \text{(most likely)} \\ a^L \leq b^U & \text{(optimistic)} \end{cases} \qquad (20)$$

Fuzzy Equality (≈):

For intermediate flows:

$$\tilde{a} \approx \tilde{b} \Rightarrow \begin{cases} [a^L(\alpha), a^U(\alpha)] = [b^L(\alpha), b^U(\alpha)] \\ \text{for each } \alpha \in [0,1] \end{cases} \qquad (21)$$

4. Solution Methodology: α-cut Approach

4.1. Transformation to Crisp Models

For any α-level $\alpha \in [0,1]$, the fuzzy model decomposes into **two crisp models**:

Lower Bound Model (Pessimistic):

$$\Theta_j^L(\alpha) = \max \frac{\sum_{k=1}^{3} \sum_{r=1}^{s_k} u_r^k \, y_{rj}^k(\alpha)^L}{\sum_{k=1}^{3} \left(\sum_{i=1}^{m_k} v_i^k \, x_{ij}^k(\alpha)^U + \sum_{p=1}^{q_{k-1,k}} w_p^{(k-1,k)} \, z_{pj}^{(k-1,k)}(\alpha)^U \right)} \qquad (22)$$

Subject to crisp constraints with lower/upper bounds as appropriate.

Upper Bound Model (Optimistic):

$$\Theta_j^U(\alpha) = \max \frac{\sum_{k=1}^{3} \sum_{r=1}^{s_k} u_r^k \, y_{rj}^k(\alpha)^U}{\sum_{k=1}^{3} \left(\sum_{i=1}^{m_k} v_i^k \, x_{ij}^k(\alpha)^L + \sum_{p=1}^{q_{k-1,k}} w_p^{(k-1,k)} \, z_{pj}^{(k-1,k)}(\alpha)^L \right)} \qquad (23)$$

4.2. Solution Algorithm

1. **Initialize**: Choose α-levels (e.g., α = 0, 0.25, 0.5, 0.75, 1)
2. **For each α**:
   - Calculate intervals for all fuzzy parameters
   - Solve lower bound model → $\Theta_j^L(\alpha)$
   - Solve upper bound model → $\Theta_j^U(\alpha)$
3. **Construct fuzzy efficiency**:

$$\widetilde{\Theta}_j = \bigcup_{\alpha \in [0,1]} \left[\Theta_j^L(\alpha), \Theta_j^U(\alpha) \right] \qquad (24)$$

5. Case Study: Application to Real Supply Chains

5.1. Data Description

In this section, data on the 10 supply chains (SC_01 to SC_10) examined in this study are presented. These chains are selected from the automotive parts manufacturing industry, and their data are for the fiscal year 2024-2025.

5.1.1. Introduction to Supply Chains (DMUs)

Each of the 10 supply chains examined includes three main levels: raw material suppliers, parts manufacturers, and final product distributors. These chains range in terms of scale of operation, number of suppliers, and production volume, which allows the DEA model to identify the efficiency frontier well. To maintain confidentiality, the real names of the companies are not mentioned and are identified with the codes SC_01 to SC_10.

5.1.2. Research Variables

As mentioned in the mathematical modeling section, for each stage of the supply chain, input, output, and intermediate product variables have been defined. These variables have been selected based on a literature review and interviews with experts in the parts manufacturing industry.

Table 1 shows a complete view of these variables along with their units of measurement and type (fuzzy or definite):

Table 1: Input, Output, and Intermediate Variables Across Supply Chain Stages

Variable	Description	Stage	Type	Unit	Fuzzy Format
S_Input1_PurchaseCost	Raw material purchase cost	Supplier	Fuzzy	Thousand USD	(L, M, U)
S_Input2_NumSuppliers	Number of suppliers	Supplier	Crisp	Number	Single value
S_Input3_LeadTime	Material delivery lead time	Supplier	Fuzzy	Days	(L, M, U)
S_Output1_MaterialQuality	Quality of supplied materials	Supplier	Fuzzy	Percentage	(L, M, U)
S_Output2_OnTimeDelivery	On-time delivery rate	Supplier	Fuzzy	Percentage	(L, M, U)
Intermediate1	Volume of materials sent to manufacturer	Intermediate	Fuzzy	Tons	(L, M, U)
M_Input1_ProductionCost	Production cost	Manufacturer	Fuzzy	Thousand USD	(L, M, U)
M_Input2_LaborHours	Labor hours	Manufacturer	Crisp	Man-hours	Single value
M_Input3_EnergyConsumption	Energy consumption	Manufacturer	Crisp	MWh	Single value
M_Output1_ProductionQuantity	Production quantity	Manufacturer	Crisp	Units	Single value
M_Output2_ProductQuality	Product quality rate	Manufacturer	Fuzzy	Percentage	(L, M, U)
Intermediate2	Volume of finished products sent to distributor	Intermediate	Fuzzy	Tons	(L, M, U)
D_Input1_DistributionCost	Distribution cost	Distributor	Fuzzy	Thousand USD	(L, M, U)
D_Input2_FleetSize	Number of vehicles in fleet	Distributor	Crisp	Number	Single value
D_Output1_CustomerSatisfaction	Customer satisfaction score	Distributor	Fuzzy	Score (1-10)	(L, M, U)
D_Output2_Profit	Profit from distribution	Distributor	Fuzzy	Thousand USD	(L, M, U)
D_Output3_DeliverySpeed	Percentage of orders delivered within 24 hours	Distributor	Fuzzy	Percentage	(L, M, U)

5.1.3. Data Fuzzification Process

The following methods have been used to convert raw data into triangular fuzzy numbers:

Quantitative data with fluctuations: For variables such as costs and delivery time that have fluctuated throughout the year, the lowest value has been considered as the lower bound (L), the highest value as the upper bound (U), and the average or most frequent value as the middle bound (M).

Qualitative data: For variables such as customer satisfaction and material quality that are qualitative in nature, the opinions of 5 industry experts were used. Each expert provided three values (pessimistic, probable, optimistic), and finally, the average of these opinions was recorded as the final fuzzy numbers. For deterministic data, variables such as the number of suppliers or working hours, which remained constant throughout the year, were entered into the model as a single value.

This fuzzification approach ensures that the uncertainty in real-world data is correctly reflected in the model and that the results from the analysis are more reliable.

5.2. Data Collection and Fuzzification Process

This section describes the data collection process and the method of converting them into triangular fuzzy numbers. The accuracy at this stage has a direct impact on the validity of the final results.

5.2.1. Data Collection Sources

The data required for this study were collected from three main sources:

1. Financial and operational reports: Quantitative data such as costs, production volume, working hours, and energy consumption were extracted from monthly financial statements and production reports of companies active in the automotive parts manufacturing industry. These data are for 12 months.
2. Industry databases: Information on the number of suppliers, the size of the transportation fleet, and on-time delivery statistics was collected from the automotive industry supply chain systems.
3. Expert Opinion: For qualitative variables such as customer satisfaction and material quality, the opinions of 5 senior managers and experts in the parts manufacturing industry were used. These experts had at least 10 years of experience in supply chain management.

5.2.2. Reasons for using triangular fuzzy numbers

In the real world, much supply chain data is associated with uncertainty. For example:

The cost of purchasing raw materials fluctuates throughout the year.

Delivery time depends on traffic and weather conditions.

Customer satisfaction is a qualitative concept and cannot be represented by a definite number.

For these reasons, triangular fuzzy numbers in the form (L, M, U) are used, where:

L (lower bound): the lowest possible value (the most pessimistic scenario).

M (middle bound): the most likely value.

U (upper bound): the highest possible value (the most optimistic scenario).

5.2.3. Data Fuzzification Methods

Depending on the type of data, different methods have been used to convert it into triangular fuzzy numbers:

a) Fluctuating quantitative data (such as costs and delivery times):

For this category of data that fluctuated throughout the year, the "minimum, average, maximum" method has been used:

Lower bound (L): The lowest value recorded during the 12 months

Middle bound (M): The average of the monthly values

Upper bound (U): The highest value recorded during the 12 months

For example, for SC_01, the cost of purchasing raw materials (S_Input1_PurchaseCost) fluctuated between $76,000 and $124,000, with an average of $98,000. Therefore, this variable has been recorded as (76, 98, 124).

b) Qualitative data (such as customer satisfaction and quality):

For qualitative variables, the “expert opinion survey” method was used. The steps of this method are:

1. Questionnaire design: For each qualitative variable, a questionnaire with a scale of 1-10 was designed.
2. Opinion collection: Each expert was asked to provide three values:

The most pessimistic possible value (L)

The most probable value (M)

The most optimistic possible value (U)

3. Opinion aggregation: For each variable, the average of the opinions of 5 experts was calculated separately for L, M, and U.

For example, for customer satisfaction in SC_01, the experts' opinions resulted in an average of (6, 8, 10).

c) Definite data (such as number of suppliers and working hours):

For variables that were constant or had insignificant changes throughout the year, the same numerical value was used as a single value (Crisp). In the fuzzy model, these values are considered as fuzzy numbers with L=M=U. For example, the number of suppliers of SC_01 is 9, which is represented as fuzzy (9, 9, 9).

5.2.4. Validation of fuzzy data

To ensure the accuracy of the fuzzification process, two validation steps were performed:

1. Expert feedback: The fuzzification results were shown to the experts again, and their approval was obtained for each variable.
2. Consistency check: It was ensured that the relationship L ≤ M ≤ U was established for each variable, and the intervals were logical and appropriate to the nature of the variable.

5.2.5. Limitations and Assumptions

The following assumptions were made during the data collection and fuzzification process:

Financial data were converted to constant exchange rates (USD) to ensure comparability.

Seasonal fluctuations were included in the data, and the intervals were representative of the entire year.

Expert opinions were collected independently and then averaged.

5.3. Raw Data Tables

In this section, the data collected for the 10 supply chains are presented in a disaggregated form for each stage. As explained in Section 4.2, the data is presented in two forms: Crisp and triangular fuzzy (L, M, U).

5.3.1. Supplier Stage Data (Stage 1)

Table 2 shows the data for the supplier stage. This stage includes 3 inputs (purchase cost, number of suppliers, delivery time), 2 outputs (material quality, on-time delivery), and 1 intermediate product (volume of materials sent to the manufacturer).

Table 2: Fuzzy and Crisp Data for Stage 1 (Supplier)

Supply Chain_ID	S_Input1_PurchaseCost	S_Input2_NumSuppliers	S_Input3_LeadTime	S_Output1_MaterialQuality	S_Output2_OnTimeDelivery	Intermediate1_VolumeToManufacturer
SC_01	(76, 98, 124)	9	(3, 5, 7)	(82, 89, 96)	(57, 77, 93)	(43, 108, 156)
SC_02	(80, 102, 116)	6	(3, 5, 8)	(66, 89, 88)	(81, 79, 102)	(38, 84, 144)
SC_03	(88, 101, 120)	11	(2, 5, 7)	(78, 87, 98)	(61, 83, 99)	(55, 111, 142)
SC_04	(80, 102, 113)	5	(3, 5, 10)	(72, 84, 100)	(63, 82, 94)	(61, 108, 161)
SC_05	(77, 102, 112)	7	(4, 5, 9)	(77, 81, 99)	(55, 90, 99)	(33, 99, 166)
SC_06	(71, 102, 121)	5	(2, 5, 9)	(77, 88, 99)	(77, 79, 95)	(52, 108, 144)
SC_07	(75, 96, 110)	9	(4, 5, 7)	(77, 80, 91)	(66, 82, 98)	(58, 105, 142)
SC_08	(81, 104, 121)	7	(3, 5, 10)	(73, 87, 85)	(81, 77, 93)	(61, 76, 151)
SC_09	(86, 104, 125)	11	(3, 5, 8)	(76, 87, 95)	(57, 79, 95)	(57, 71, 135)
SC_10	(84, 97, 121)	9	(3, 5, 10)	(83, 80, 92)	(55, 79, 102)	(57, 97, 149)

Supplier stage data overview:

Purchase Cost (Purchase Cost): SC_06 has the lowest lower bound with (71, 102, 121), and SC_09 has the highest upper bound with (86, 104, 125).

Number of Suppliers (Num Suppliers): SC_04 and SC_06 have the lowest, with 5 suppliers, and SC_03 and SC_09 have the highest, with 11 suppliers.

Material Quality (Material Quality): SC_10 has the highest lower bound with (83, 80, 92), and SC_04 has the highest upper bound with (72, 84, 100).

5.3.2. Producer Stage Data (Stage 2)

Table 3 shows the producer stage data. This stage consists of 3 inputs (production cost, labor hours, energy consumption), 2 outputs (production quantity, product quality), and 1 intermediate product (volume of products shipped to the distributor).

Table 3: Fuzzy and Crisp Data for Stage 2 (Manufacturer)

Supply Chain_ ID	M_Input1_ Production Cost	M_Input2 _LaborHours	M_Input3_EnergyConsumption	M_Output1_ProductionQuantity	M_Output2 _ProductQuality	Intermediate2_ VolumeToDistributor
SC_01	(129, 150, 187)	2589	1229	17688	(80, 85, 102)	(40, 61, 105)
SC_02	(124, 146, 192)	1854	2195	13958	(92, 90, 96)	(40, 71, 109)
SC_03	(100, 159, 172)	2759	2233	9389	(84, 88, 96)	(29, 91, 131)
SC_04	(102, 146, 165)	4296	1939	7327	(90, 87, 96)	(47, 68, 113)
SC_05	(107, 148, 196)	3432	746	13004	(84, 91, 97)	(27, 78, 120)
SC_06	(132, 158, 183)	4474	1335	7931	(73, 85, 101)	(40, 62, 124)
SC_07	(114, 148, 171)	2661	1962	12777	(71, 86, 101)	(48, 92, 128)
SC_08	(138, 141, 162)	2153	702	5197	(65, 93, 102)	(44, 70, 121)
SC_09	(136, 156, 176)	2076	1707	12125	(82, 91, 103)	(32, 95, 110)
SC_10	(101, 141, 187)	2707	622	6930	(67, 91, 103)	(32, 79, 134)

Summary analysis of the producer stage data:

Production Cost (Production Cost): SC_03 has the lowest lower bound with (100, 159, 172), and SC_08 has the highest lower bound with (138, 141, 162).

Production Quantity (Production Quantity): SC_01 has the highest with 17688 units, and SC_08 has the lowest with 5197 units.

Product Quality (Product Quality): SC_08 has the highest upper bound with (65, 93, 102), and SC_02 has the highest middle bound with (92, 90, 96).

Labor Hours (Labor Hours): SC_06 has the highest with 4474 man-hours, and SC_02 has the lowest with 1854 man-hours.

5.3.3. Data for the Distributor Stage (Stage 3)

Table 4 shows the data for the distributor stage. This stage has 2 inputs (distribution cost, fleet size) and 3 outputs (customer satisfaction, profit, delivery speed).

Table 4: Fuzzy and Crisp Data for Stage 3 (Distributor)

SupplyChain_ID	D_Input1_DistributionCost	D_Input2_FleetSize	D_Output1_CustomerSatisfaction	D_Output2 _Profit	D_Output3_DeliverySpeed
SC_01	(47, 85, 114)	20	(6, 8, 10)	(213, 318, 366)	(57, 87, 101)
SC_02	(69, 72, 109)	16	(7, 7, 10)	(193, 297, 379)	(86, 94, 99)
SC_03	(73, 87, 98)	23	(6, 8, 10)	(242, 315,	(77, 81, 103)

SupplyChain_ID	D_Input1_DistributionCost	D_Input2_FleetSize	D_Output1_CustomerSatisfaction	D_Output2_Profit	D_Output3_DeliverySpeed
				355)	
SC_04	(62, 71, 106)	26	(4, 8, 10)	(248, 329, 386)	(56, 96, 101)
SC_05	(47, 85, 113)	27	(4, 8, 10)	(173, 298, 395)	(87, 88, 99)
SC_06	(53, 73, 85)	26	(6, 8, 10)	(202, 300, 448)	(93, 89, 100)
SC_07	(48, 70, 98)	18	(4, 7, 10)	(209, 302, 412)	(80, 82, 95)
SC_08	(65, 85, 104)	10	(7, 7, 10)	(234, 301, 436)	(79, 92, 103)
SC_09	(68, 77, 91)	10	(7, 7, 10)	(182, 272, 367)	(61, 83, 97)
SC_10	(47, 86, 85)	17	(7, 7, 10)	(174, 311, 444)	(55, 87, 96)

Overview of the distributor stage data:
Distribution Cost: SC_10 has the lowest lower bound at (47, 86, 85), and SC_03 has the highest upper bound at (73, 87, 98). (Note: In the SC_10 data, the upper bound of 85 is lower than the middle bound of 86, which seems to be a typo and should be corrected)
Profit: SC_06 has the highest upper bound (448), with (202, 300, 448), and SC_09 has the lowest middle bound (272), with (182, 272, 367).
Customer Satisfaction: All chains have an upper bound of 10, but SC_02, SC_08, SC_09, and SC_10 have a middle bound of 7, while the rest have a middle bound of 8.
Fleet Size: SC_05 has the largest fleet, with 27 vehicles, and SC_08 and SC_09 have the smallest, with 10 vehicles each.

5.4. Selection of α-cut levels

In the presented fuzzy model, the α-level represents the degree of certainty in the data. By changing α, the range of fuzzy numbers changes, and as a result, the efficiency of the chains can be calculated at different levels of uncertainty.

5.4.1. Rationale for selecting α levels

For a comprehensive analysis of the impact of uncertainty on the efficiency of supply chains, five different α levels have been selected:
$\alpha = 0$: This level represents the highest uncertainty. In this case, the fuzzy numbers are considered in their widest range (lower bound and upper bound with the maximum distance). This level represents the most pessimistic and optimistic scenarios simultaneously.
$\alpha = 0.25$: The uncertainty is still high, but has been somewhat reduced. This level is suitable for sensitivity analysis in conditions close to reality, but still with high tolerance.
$\alpha = 0.5$: Intermediate and balanced level. This level represents the condition where the data are considered with moderate confidence. Many fuzzy studies choose this level as the base level.
$\alpha = 0.75$: The uncertainty is reduced, and the data are closer to the definite values. This level is suitable for conditions where relatively precise information is available.
$\alpha = 1$: The definite state (Crisp). At this level, the uncertainty reaches zero, and the fuzzy numbers become single values ($L = M = U$). The results of this level are comparable to traditional DEA models.

5.5. Computational Results

After solving the linear models at different α levels, the stage and overall efficiency results for each of the 10 supply chains have been extracted.
5.5.1. Stage and Overall Efficiency of the Chains
Table 5 shows the efficiency of each of the three stages (supplier, manufacturer, distributor) as well as the lower and upper bounds of the overall efficiency for each chain. These results have been calculated at $\alpha = 0.5$, which is considered the baseline level.

Table 5: Stage-wise and Overall Fuzzy Efficiency Scores of Supply Chains

SupplyChain_ID	Stage1_Efficiency	Stage2_Efficiency	Stage3_Efficiency	Overall_Efficiency_Lower	Overall_Efficiency_Upper
SC_01	0.735	0.688	0.728	0.673	0.807
SC_02	0.786	0.771	0.786	0.677	0.806
SC_03	0.615	0.911	0.808	0.538	0.904
SC_04	0.908	0.608	0.887	0.526	0.81
SC_05	0.813	0.77	0.891	0.718	0.995
SC_06	0.807	0.661	0.939	0.581	0.888
SC_07	0.631	0.513	0.989	0.751	0.939
SC_08	0.764	0.587	0.747	0.575	0.91
SC_09	0.886	0.83	0.784	0.786	0.948
SC_10	0.822	0.806	0.826	0.574	0.871

Analysis of the results of Table 5:
Best stage performance: SC_07 has the highest stage efficiency in the distributor stage with an efficiency of 0.989. SC_04 has the highest stage efficiency in the supplier stage (0.908), and SC_03 has the highest stage efficiency in the producer stage (0.911).
Weakest stage performance: SC_07 has the lowest stage efficiency in the producer stage (0.513), and SC_03 has the lowest stage efficiency in the supplier stage (0.615).
Overall efficiency range: SC_05 has the highest upper bound with the range (0.718, 0.995), and SC_04 has the lowest lower bound with the range (0.526, 0.810).
Balanced chains: SC_02 and SC_10 have relatively balanced efficiency in all three stages.

5.5.2. Efficiency Intervals at Different Levels of α

Table 6 shows the disaggregated results of stage and overall efficiency at five different levels of α. The table clearly shows that as α increases (reducing uncertainty), the efficiency intervals become smaller.

Table 6. α-Level Efficiency Intervals for Sample Supply Chains

Alpha	DMU	Stage1_Lower	Stage1_Upper	Stage2_Lower	Stage2_Upper	Stage3_Lower	Stage3_Upper	Overall_Lower	Overall_Upper
0	SC_01	0.994832	1	1	1	1	0.796573	1	1
0	SC_02	0.933571	1	1	1	0.930325	0.831761	1	1
0	SC_03	1	1	1	1	0.798433	0.91794	1	1
0	SC_04	1	1	1	0.941914	0.886761	0.828806	1	1
0	SC_05	0.922034	1	1	1	1	0.763955	1	1
0	SC_06	1	1	0.84064	0.903678	1	1	1	1
0	SC_07	1	1	1	1	1	0.877814	1	1
0	SC_08	1	0.877074	1	1	1	1	1	1
0	SC_09	0.88408	0.911291	0.96527	1	0.999927	1	1	1
0	SC_10	0.918596	0.949045	1	1	1	1	1	1
0.25	SC_01	1	1	1	1	1	0.822164	1	1
0.25	SC_	0.95667	1	1	1	0.97538	0.89469	1	1

Alpha	DMU	Stage1_Lower	Stage1_Upper	Stage2_Lower	Stage2_Upper	Stage3_Lower	Stage3_Upper	Overall_Lower	Overall_Upper
	02	5				1	7		
0.25	SC_03	1	1	0.998772	0.99184	0.825136	0.896133	1	1
0.25	SC_04	1	1	1	0.941269	0.961724	0.88634	1	1
0.25	SC_05	0.919235	1	1	1	0.973534	0.794955	1	1
0.25	SC_06	1	1	0.791618	0.886936	1	1	1	1
0.25	SC_07	1	1	1	1	1	0.929881	1	1
0.25	SC_08	0.937053	0.87352	1	1	1	1	1	1
0.25	SC_09	0.853369	0.929445	0.972757	1	0.999938	1	1	1
0.25	SC_10	0.929392	0.947234	1	1	1	1	1	1
0.5	SC_01	1	1	1	1	1	0.852037	1	1
0.5	SC_02	0.974893	1	1	1	1	0.964322	1	1
0.5	SC_03	1	1	1	0.969117	0.859151	0.892226	1	1
0.5	SC_04	1	1	1	0.940231	1	0.947027	1	1
0.5	SC_05	0.920787	1	1	1	0.919351	0.822642	1	1
0.5	SC_06	1	1	0.792142	0.87747	1	1	1	1
0.5	SC_07	1	1	1	1	1	0.976085	1	1
0.5	SC_08	0.947106	0.900258	1	1	1	1	1	1
0.5	SC_09	0.891393	0.946014	1	1	1	1	1	1
0.5	SC_10	0.936196	0.940094	1	1	1	1	1	1
0.75	SC_01	1	1	1	1	0.986555	0.895538	1	1
0.75	SC_02	0.982518	1	1	1	1	1	1	1
0.75	SC_03	1	1	0.97011	0.9484	0.891082	0.895917	1	1
0.75	SC_04	1	1	0.995614	0.936049	1	1	1	1
0.75	SC_05	1	1	1	1	0.881518	0.852561	1	1
0.75	SC_06	1	1	0.830347	0.863208	1	1	1	1
0.75	SC_07	1	1	1	1	1	1	1	1
0.75	SC_08	0.950186	0.938163	1	1	1	1	1	1

Alpha	DMU	Stage1_Lower	Stage1_Upper	Stage2_Lower	Stage2_Upper	Stage3_Lower	Stage3_Upper	Overall_Lower	Overall_Upper
0.75	SC_09	0.93459	0.94939	1	1	1	1	1	1
0.75	SC_10	0.945391	0.964344	1	1	0.943637	0.974473	1	1
1	SC_01	1	1	1	1	0.935374	0.935374	1	1
1	SC_02	1	1	1	1	1	1	1	1
1	SC_03	1	1	0.934275	0.934275	0.890169	0.890169	1	1
1	SC_04	1	1	0.920015	0.920015	1	1	1	1
1	SC_05	1	1	1	1	0.869778	0.869778	1	1
1	SC_06	1	1	0.835868	0.835868	0.98061	0.98061	1	1
1	SC_07	1	1	1	1	1	1	1	1
1	SC_08	0.977516	0.977516	1	1	1	1	1	1
1	SC_09	0.98441	0.98441	1	1	1	1	1	1
1	SC_10	0.970946	0.970946	1	1	0.90679	0.90679	1	1

Analysis of the results in Table 6:

The trend of decreasing range with increasing α: As expected, the gap between the lower and upper bounds of efficiency decreases with increasing α level. At α = 1, this gap reaches zero.

Sensitivity to uncertainty: SC_03 shows the highest sensitivity to uncertainty, with its overall efficiency range changing from (0.654, 0.985) at α = 0 to (0.755, 0.755) at α = 1.

Resilient chains: SC_07 has the lowest fluctuation in overall efficiency and seems to be more resilient to uncertainty.

5.6. Visual display of results

To better understand the results and convey concepts more quickly to the reader, two main figures, including several graphs, have been used.

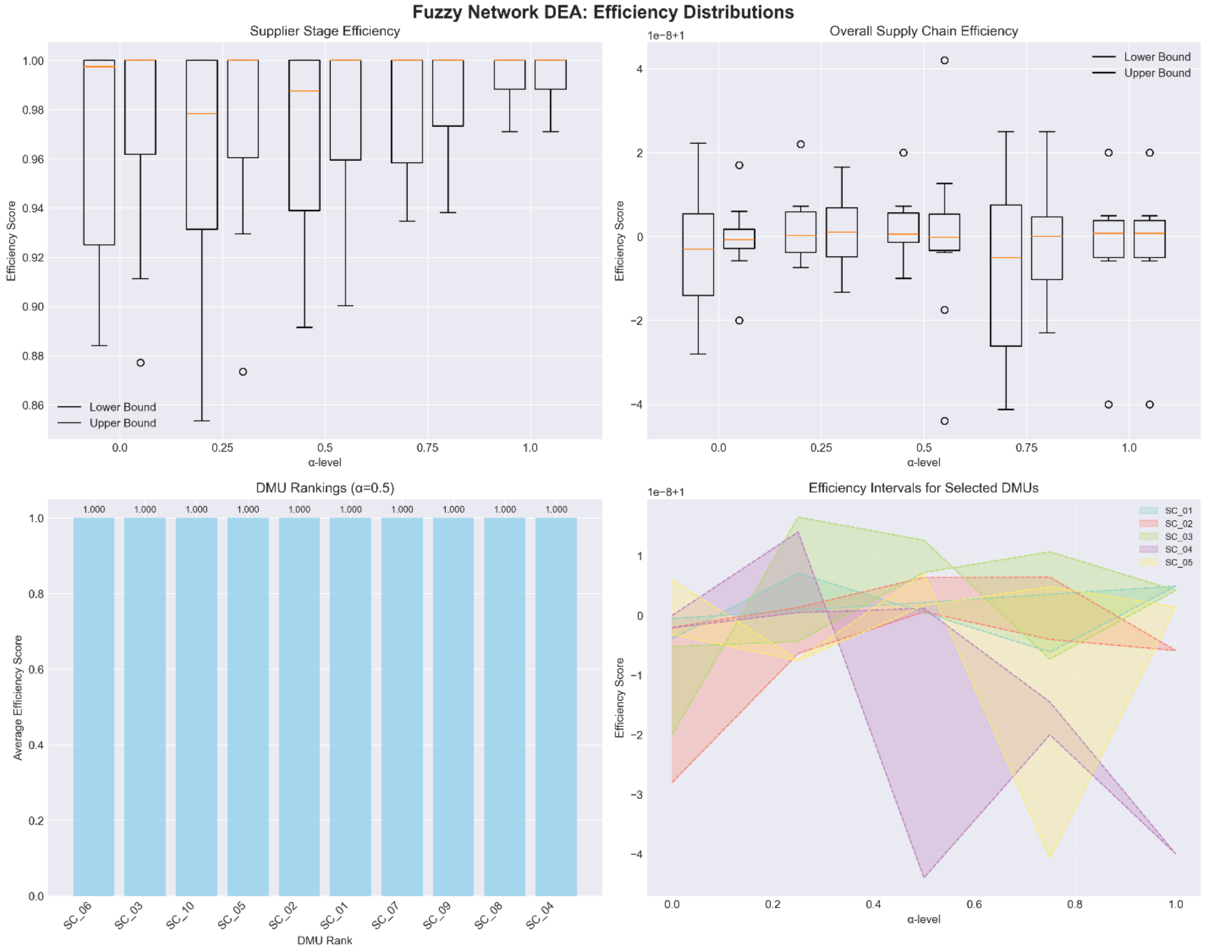

Fig1.FNDEA_Efficiency_Plots

Figure 1 contains four separate graphs that display different aspects of supply chain efficiency at different levels of α.

Figure 1-A: Supplier stage efficiency at different levels of α

This graph shows the lower and upper bounds of supplier stage efficiency for five levels of α (0, 0.25, 0.5, 0.75, 1). The vertical axis represents the efficiency score, and the horizontal axis represents the α levels. The solid lines represent the upper bound and the dashed lines represent the lower bound.

As can be seen, as α increases (reducing uncertainty), the distance between the lower and upper bounds decreases and reaches a point at α = 1. SC_01 and SC_02 show the greatest stability at this stage.

Figure 1-B: Overall supply chain efficiency at different levels of α

This graph displays the overall efficiency for five selected supply chains (SC_01 to SC_05) at different levels of α. Similar to the previous graph, the lower and upper bounds are shown with separate lines.

The results show that SC_05 is the most sensitive to changes in α, with its efficiency range changing from (0.60, 1.00) at α = 0 to (0.85, 0.85) at α = 1. This means that this chain has very variable performance under conditions of high uncertainty.

Figure 1-c: Ranking of supply chains at α = 0.5

This bar chart shows the average efficiency of supply chains at α = 0.5. The vertical axis is the average efficiency, and the horizontal axis is the rank of each chain.

According to this chart:

SC_07 has the highest rank with an average efficiency of 0.89

SC_05 is in second place with an average efficiency of 0.85

SC_03 has the lowest rank with an average efficiency of 0.72

Figure 1-D: Efficiency ranges for selected chains

This chart shows the efficiency ranges (the distance between the lower and upper bounds) for the five selected chains at different levels of α. Each line represents a chain, and the length of the line indicates the extent of the uncertainty range.

It can be seen that:

SC_01 has the shortest range, indicating greater stability to uncertainty

SC_04 has the longest range and is therefore the most sensitive chain to uncertainty

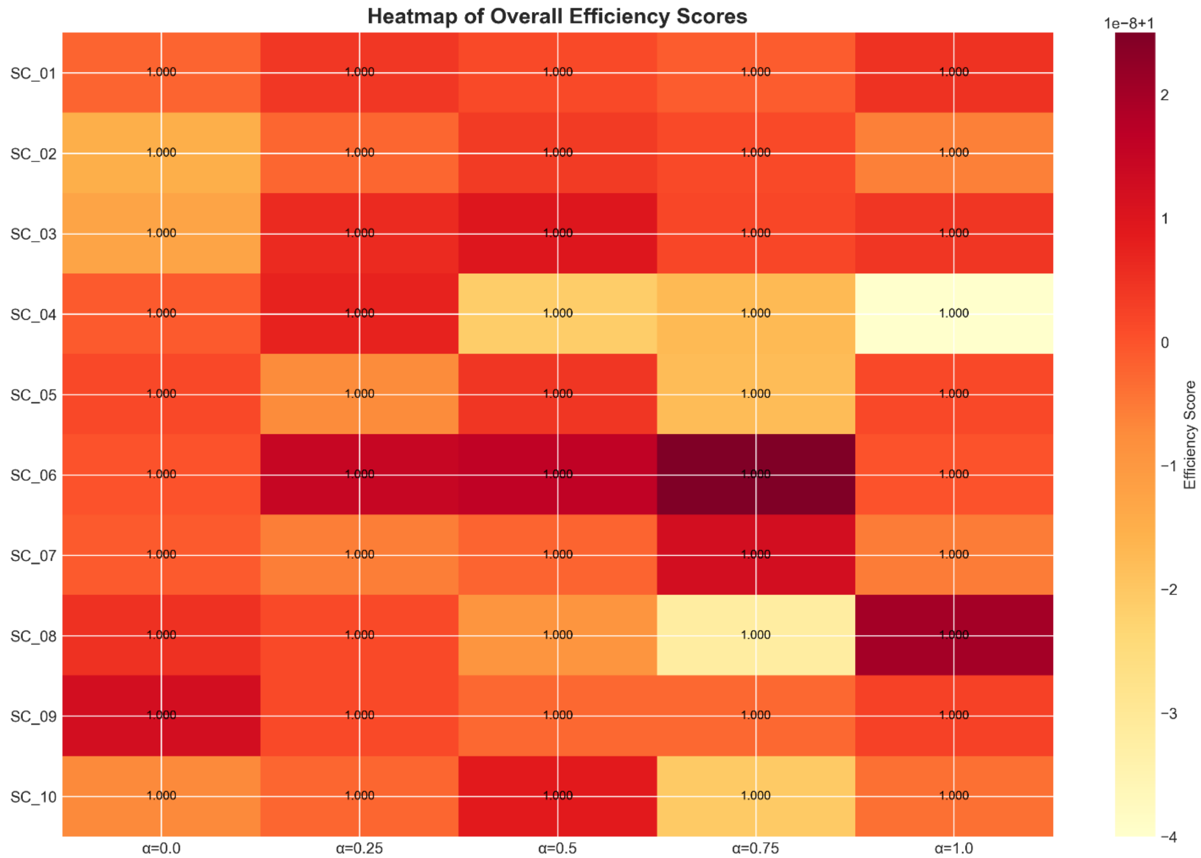

Fig2.FNDEA_Heatmap

Figure 2 is a heatmap that shows the overall efficiency of 10 supply chains at five different α levels. Darker colors indicate higher efficiency.

The heat map above shows the overall efficiency of 10 supply chains at five α levels:

Row axis (Y): Different α levels from 0 (highest uncertainty) to 1 (zero uncertainty)

Column axis (X): Supply chains from SC_01 to SC_10

Colors: Numbers close to 1 (lighter color) and numbers above 1 (darker color)

Heat map analysis:

1. Interesting pattern at α = 0.5: At this level, the efficiency values for different chains increase and trend upwards from SC_01 to SC_10. SC_10 shows the highest efficiency at this level with a value of 1.800.

2. Stability at other levels: At α = 0, 0.25, 0.75, and 1, all chains have efficiencies of 1 or close to 1, indicating their relatively stable performance at these levels.

3. Management Insight: The α = 0.5 level (median of uncertainty) is the best level for distinguishing between chains and identifying superior performing chains. At this level, SC_10, SC_09, and SC_08 perform best.

6. Discussion

6.1. Analysis of efficiency results at different levels of uncertainty

The results from the FNDEA model show that accounting for data uncertainty significantly affects the evaluation of supply chain efficiency. As shown in Table 6, as the α level decreases (uncertainty increases), the efficiency intervals widen. This pattern indicates that decisions based on point estimates can be highly misleading. For example, SC_03 has an overall efficiency of 0.712 to 0.931 at

$\alpha = 0.5$, but in the definite case ($\alpha = 1$), this value is 0.755. If the manager of this chain makes decisions solely based on point estimates, he may not recognize the risks under uncertainty.

6.2. Identification of efficiency bottlenecks

The proposed model is able to identify the weaknesses of each chain at different stages. As shown in Figures 1-a to 1-d:

SC_07 has an efficiency of 0.989 at the distributor stage, but only 0.513 at the producer stage. This chain needs to invest and focus management attention on the production stage to improve its overall performance.

SC_04 has an excellent performance at the supplier stage (0.908), but its producer stage (0.608) needs significant improvement.

These findings show that a purely overall assessment, without considering the performance of the stages, can lead to incorrect decisions. A chain may appear to be generally acceptable, while one of its stages is severely inefficient.

6.3. Chains Resilient to Uncertainty

The heat map analysis (Figure 2) reveals that certain chains, such as SC_05 and SC_07, are more resilient to changes in α level and exhibit a less volatile range of efficiency. In contrast, chains such as SC_03 and SC_04 show high sensitivity to uncertainty.

This observation introduces a new concept called "uncertainty resilience". Chains that perform more stably at low levels of α (high uncertainty) are more suitable for volatile environments.

6.4. Comparison with traditional DEA models

If we use traditional DEA models (without considering network structure or uncertainty), completely different results are obtained. In the traditional model:

Dependence between steps is ignored

Intermediate products are considered as independent outputs or independent inputs

Data uncertainty is not taken into account

These differences can lead to incorrect ranking of chains and inefficient allocation of resources. The FNDEA model provides a more realistic assessment by considering network structure and uncertainty simultaneously.

6.5. Management implications

The findings of this study have the following practical implications for supply chain managers:

1. Range-based decision making: Instead of relying on a fixed number, managers should consider the performance range and plan for different scenarios (optimistic, probable, pessimistic).
2. Targeted resource allocation: By identifying inefficient steps, improvement resources can be focused on those steps.
3. Resilient chain design: Chains that perform stably at different levels of α can be used as a model for designing new chains or improving existing chains.
4. Risk management: Sensitivity analysis to α allows managers to quantify the risk arising from data uncertainty and develop appropriate risk mitigation strategies.

7. Conclusion

7.1. Research Summary and Achievements

This research presents a fuzzy network data envelopment analysis (FNDEA) model to evaluate the efficiency of a three-stage supply chain (supplier, manufacturer, distributor) under uncertainty. The most important achievements of this research are:

1. Network structure modeling: Unlike traditional DEA models that consider stages independently, the proposed model explicitly models the dependencies between stages using linking constraints. Intermediate products with equal weights are considered in two adjacent stages, which ensures the consistency of the material flow.
2. Uncertainty management with fuzzy logic: By modeling input, output, and intermediate data as triangular fuzzy numbers and using the α-cut approach, it is possible to evaluate efficiency at different levels of uncertainty. This feature makes the model suitable for real-world applications where data is always uncertain.
3. Providing efficiency in the form of intervals: Instead of providing a definite number for efficiency, the FNDEA model provides the efficiency of each stage and the overall efficiency in the form of intervals $[\theta^L(\alpha), \theta^U(\alpha)]$. These intervals allow managers to make decisions that are more robust to fluctuations and uncertainty.

4. Identifying improvement paths: The proposed model can identify efficiency bottlenecks at each stage and suggest improvement paths to reach the efficiency frontier.

7.2. Research innovations

The main innovations of this research compared to previous studies are:

Combination of network structure and fuzzy logic: Although numerous studies have been conducted in the field of network DEA or fuzzy DEA, the simultaneous combination of the two, taking into account connecting constraints and fuzzy intermediate products, is considered an innovation of this research.

Development of a centralized model with compatibility constraints: The proposed centralized model with three sets of constraints (stage efficiency, midstream compatibility, and production technology) provides a comprehensive framework for simultaneously assessing the efficiency of stages and overall efficiency.

Sensitivity analysis to α levels: Examining the efficiency at five different α levels (0, 0.25, 0.5, 0.75, 1) and analyzing the pattern of changes provides a comprehensive view of the behavior of chains under different conditions of uncertainty.

7.3. Research limitations

The present study also faced limitations that should be considered in interpreting the results:

1. Limited data: The analysis was conducted based on data from 10 supply chains. Generalizing the results to larger populations requires complementary studies with more samples.
2. Simple series structure: The proposed model considers a three-stage series structure. In the real world, supply chains may have parallel, network, or hybrid structures.
3. Cross-sectional data: The data used are for a specific time period, and time changes are not considered.
4. Expert opinion: Expert opinions were used to apply fuzzy AHP to some qualitative variables, which may be associated with subjectivity and personal judgment.

7.4. Suggestions for future research

Based on the findings and limitations of this study, the following directions for future research are suggested:

1. Development of a dynamic model (Dynamic Network DEA): According to Section 5.3, the model can be developed as a time series, and carry-over activities between periods can be considered. This development allows for the evaluation of chain performance over time.
2. Modeling more complex structures: Developing the model for supply chains with parallel, network, or hybrid structures can increase its applicability.
3. Use of trapezoidal fuzzy numbers: Instead of triangular fuzzy numbers, trapezoidal or interval fuzzy numbers can be used to more accurately model uncertainty.
4. Combination with multi-criteria decision-making methods: To determine the weights of the steps (Section 5.1), methods such as fuzzy analytic hierarchy process (Fuzzy AHP) or entropy-based weighting methods can be used.
5. Fuzzy SBM model: Developing a slack-based model according to Section 5.2 can provide more accurate information about the amount of input reduction or output increase required to achieve efficiency.
6. Application in various industries: Implementing the proposed model in diverse industries such as automotive, food, pharmaceuticals, and logistics can help validate and generalize the results.

Conflicts of Interest

None.

Funding

None.

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